MATH 112A Exam Review Combinations and Permuatations

1. Evaluate each of the following

a) $7!$ b) $\frac{80!}{78!}$ c) $2! \times 4!$ d) $\frac{3! * 5!}{4!}$

2. Write as a ratio (fraction) of factorials

a) 22 x 21 x 20 x 19 b) 18 x 17 x 16 x 4 x 3

3. Write each as its factorial ratio equivalent then evaluate each of the following

a) ${}_6P_4$ b) ${}_3P_2$ c) ${}_5C_2$ d) ${}_{10}C_4$

4. A club has 10 members

a) In how many ways can a president, vice president and secretary be elected?
b) In how many ways can a committee of 4 people be elected?

5. In how many ways can you pick 5 pairs of socks from a drawer with 8 pairs.

6. There are 10 people on a basketball team, in how many ways can the coach select a 5 man starting lineup

7. In how many ways can a had of 5 clubs be dealt from a standard deck

8. How many ways can the letters of the following words be arranged?

a) goat b) Hampton c) Mississippi d) preposterous

9. A club consists of 10 males and 15 females, how many different committees of 5 members can be formed with :

a) no restrictions b) 5 males
c) 2 males and 3 females d) all females

10. A total of 5 red, 3 blue and 4 yellow marbles are placed in a container. If 3 marbles are removed find the probability of

a) removing first a 1 red and then 2 yellow
b) removing first a red, then a blue then a yellow
c) removing 3 blue marbles.

11. Five cards are selected from a standard deck of 52 cards. What is the probability of selecting:

a) 3 tens and 2 nines (full house) b) 5 diamonds c) selecting 3 aces, 1 king and 1 Queen

12. Expand the following binomials. (Remember Pascal's triangle)

a) $(x + y)^3$ b) $(2x + y)^3$ c) $(2x - y)^4$ $(3x + 2y)^4$

13. What is the value of term 8 of the expression $(2x - 5y)^{10}$

SOLUTIONS

1.

a) 7 x 6 x 5 x 4 x 3 x 2 x 1 = 5040

b) $\frac{80*79*\cancel{78}*\cancel{77}*\cancel{76}*\ldots}{\cancel{78}*\cancel{77}*\cancel{76}\ldots} = 80*79 = 6320$

c) (2 x 1) x (4 x 3 x 2 x 1) = 48

c) $\frac{(3*2*1)(5*\cancel{4}*\cancel{3}*\cancel{2}*\cancel{1})}{(\cancel{4}*\cancel{3}*\cancel{2}*\cancel{1})} = 30$

2.

a) $\frac{22!}{18!}$

b) $\frac{18!*4!}{15!*2!}$

3. a) ${}_6P_4 \Longrightarrow \frac{6!}{2!} = 30$

b) ${}_3P_2 = \frac{3!}{1!} = 6$

c) ${}_5C_2 = \frac{5!}{2!3!} = 10$

d) ${}_{10}C_4 = \frac{10!}{4!6!} = 210$

4. a) ${}_{10}P_3 = \frac{10!}{7!} = 720$

b) ${}_{10}C_4 = \frac{10!}{4!6!} = 210$

5. ${}_8C_5 = \frac{8!}{3!5!} = 56$

6. ${}_{10}C_5 = \frac{10!}{5!5!} = 252$

7. ${}_{52}C_5 = \frac{52!}{5!47!} = 2598960$

8. a) ${}_4P_4 = 24$ b) ${}_7P_7 = 5040$ c) $\frac{11!}{2!4!4!} = 34650$ d) $\frac{12!}{2!2!2!2!2!} = 14968800$

9. a) ${}_{25}C_5 = \frac{25!}{5!20!} = 53130$

b) ${}_{10}C_5 = \frac{10!}{5!5!} = 252$

c) ${}_{10}C_2$ x ${}_{15}C_3 = \frac{(10!)(15!)}{(2!8!)(3!12!)} = 20475$

d) ${}_{15}C_5 = \frac{15!}{5!10!} = 3003$

10. a) $\frac{({}_5P_1)({}_4P_2)}{{}_{12}P_3} = \frac{60}{1320} = \frac{1}{22}$ or 0.0454545…

b) $\frac{({}_5P_1)({}_3P_1)({}_4P_1)}{{}_{12}P_3} = \frac{60}{1320} = \frac{1}{22}$ or 0.04545

c) $\frac{{}_3P_3}{{}_{12}P_3} = \frac{6}{1320} = \frac{1}{220}$ or 0.004545. . .

11. a) $\frac{({}_4C_3)({}_4C_2)}{{}_{52}C_5} = \frac{24}{2598960} = \frac{1}{108290}$ or 0.000009234 . . .

b) $\frac{{}_{13}C_5}{{}_{52}C_5} = \frac{1287}{2598960} = \frac{33}{66640} =$ or 0.0004951980792 . . .

c) $\frac{({}_4C_3)({}_4C_1)({}_4C_1))}{{}_{52}C_5} = \frac{64}{2598960} = \frac{4}{162435}$ or 0.00002462523471 . . .

12. a) $(x + y)^3$

$$\binom{3}{3}(x)^3(y)^0 + \binom{3}{2}(x)^2(y)^1 + \binom{3}{1}(x)^1(y)^2 + \binom{3}{0}(x)^0(y)^3$$

$$(1)(x^3) + (3)(x^2)(y) + (3)(x)(y^2) + (1)(y^3)$$

$$x^3 + 3x^2y + 3xy^2 + y^3$$

b) $(2x + y)^3$

$$\binom{3}{3}(2x)^3(y)^0 + \binom{3}{2}(2x)^2(y)^1 + \binom{3}{1}(2x)^1(y)^2 + \binom{3}{0}(2x)^0(y)^3$$

$$(1)(2x)^3 + (3)(2x)^2(y) + (3)(2x)(y^2) + (1)(y^3)$$

$$8x^3 + 12x^2y + 6xy^2 + y^3$$

c) $(2x - y)^4$

$$\binom{4}{4}(2x)^4(-y)^0 + \binom{4}{3}(2x)^3(-y)^1 + \binom{4}{2}(2x)^2(-y)^2 \quad \binom{4}{1}(2x)^1(-y)^3 + \binom{4}{0}(2x)^0(-y)^4$$

$$(1)(2x)^4 + (4)(2x)^3(-y) + (6)(2x)^2(-y)^2 + (4)(2x)(-y)^3 + (1)(-y)^4$$

$$16x^4 - 32x^3y + 24x^2y^2 - 8xy^3 + y^4$$

d) $(3x + 2y)^4$

$$\binom{4}{4}(3x)^4(2y)^0 + \binom{4}{3}(3x)^3(2y)^1 + \binom{4}{2}(3x)^2(2y)^2 \quad \binom{4}{1}(3x)^1(2y)^3 + \binom{4}{0}(3x)^0(2y)^4$$

$$(1)(3x)^4 + (4)(3x)^3(2y) + (6)(3x)^2(2y)^2 + (4)(3x)(2y)^3 + (1)(2y)^4$$

$$81x^4 + 108x^3y + 216x^2y^2 + 96xy^3 + 16y^4$$

13.

1 2 3 4 5 6 7 8 9 10 11

$$\binom{10}{3}(2x)^3(-5y)^7$$

$$(120)(8x^3)(-78125y^7)$$

$$-75\,000\,000\,x^3y^7$$