

1. Evaluate each of the following

a) $7!$

b) $\frac{80!}{78!}$

c) $2! \times 4!$

d) $\frac{3! \cdot 5!}{4!}$

2. Write as a ratio (fraction) of factorials

a) $22 \times 21 \times 20 \times 19$

b) $18 \times 17 \times 16 \times 4 \times 3$

3. Write each as its factorial ratio equivalent then evaluate each of the following

a) ${}_6P_4$

b) ${}_3P_2$

c) ${}_5C_2$

d) ${}_{10}C_4$

4. A club has 10 members

a) In how many ways can a president, vice president and secretary be elected?

b) In how many ways can a committee of 4 people be elected?

5. In how many ways can you pick 5 pairs of socks from a drawer with 8 pairs.

6. There are 10 people on a basketball team, in how many ways can the coach select a 5 man starting lineup

7. In how many ways can a hand of 5 clubs be dealt from a standard deck

8. How many ways can the letters of the following words be arranged?

a) goat

b) Hampton

c) Mississippi

d) preposterous

9. A club consists of 10 males and 15 females, how many different committees of 5 members can be formed with :

a) no restrictions

b) 5 males

c) 2 males and 3 females

d) all females

10. A total of 5 red, 3 blue and 4 yellow marbles are placed in a container. If 3 marbles are removed find the probability of

a) removing first a 1 red and then 2 yellow

b) removing first a red, then a blue then a yellow

c) removing 3 blue marbles.

SOLUTIONS

- 1.
- a) $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$
- b) $\frac{80 * 79 * \cancel{78} * \cancel{77} * \cancel{76} * \dots}{\cancel{78} * \cancel{77} * \cancel{76} \dots} = 80 * 79 = 6320$
- c) $(2 \times 1) \times (4 \times 3 \times 2 \times 1) = 48$
- c) $\frac{(3 * 2 * 1)(5 * 4 * \cancel{3} * \cancel{2} * 1)}{(\cancel{4} * \cancel{3} * \cancel{2} * 1)} = 30$
- 2.
- a) $\frac{22!}{18!}$
- b) $\frac{18! * 4!}{15! * 2!}$
3. a) ${}_6P_4 \implies \frac{6!}{2!} = 30$
- b) ${}_3P_2 = \frac{3!}{1!} = 6$
- c) ${}_5C_2 = \frac{5!}{2!3!} = 10$
- d) ${}_{10}C_4 = \frac{10!}{4!6!} = 210$
4. a) ${}_{10}P_3 = \frac{10!}{7!} = 720$
- b) ${}_{10}C_4 = \frac{10!}{4!6!} = 210$
5. ${}_8C_5 = \frac{8!}{3!5!} = 56$
6. ${}_{10}C_5 = \frac{10!}{5!5!} = 252$
7. ${}_{52}C_5 = \frac{52!}{5!47!} = 2598960$
8. a) ${}_4P_4 = 24$ b) ${}_7P_7 = 5040$ c) $\frac{11!}{2!4!4!} = 34650$ d) $\frac{12!}{2!2!2!2!2!} = 14968800$
9. a) ${}_{25}C_5 = \frac{25!}{5!20!} = 53130$
- b) ${}_{10}C_5 = \frac{10!}{5!5!} = 252$
- c) ${}_{10}C_2 \times {}_{15}C_3 = \frac{(10!)(15!)}{(2!8!)(3!12!)} = 20475$
- d) ${}_{15}C_5 = \frac{15!}{5!10!} = 3003$

$$10. \quad \text{a) } \frac{{}_5P_1({}_4P_2)}{{}_{12}P_3} = \frac{60}{1320} = \frac{1}{22} \text{ or } 0.0454545\dots \quad \text{b) } \frac{{}_5P_1({}_3P_1){}_4P_1)}{{}_{12}P_3} = \frac{60}{1320} = \frac{1}{22} \text{ or } 0.04545$$

$$\text{c) } \frac{{}_3P_3}{{}_{12}P_3} = \frac{6}{1320} = \frac{1}{220} \text{ or } 0.004545\dots$$

$$11. \quad \text{a) } \frac{{}_4C_3({}_4C_2)}{{}_{52}C_5} = \frac{24}{2598960} = \frac{1}{108290} \text{ or } 0.000009234\dots$$

$$\text{b) } \frac{{}_{13}C_5}{{}_{52}C_5} = \frac{1287}{2598960} = \frac{33}{66640} \text{ or } 0.0004951980792\dots$$

$$\text{c) } \frac{{}_4C_3({}_4C_1){}_4C_1)}{{}_{52}C_5} = \frac{64}{2598960} = \frac{4}{162435} \text{ or } 0.00002462523471\dots$$

$$12. \quad \text{a) } \begin{array}{c} (x + y)^3 \\ \overbrace{\quad\quad\quad} \quad \overbrace{\quad\quad\quad} \quad \overbrace{\quad\quad\quad} \quad \overbrace{\quad\quad\quad} \\ \binom{3}{3}(x)^3(y)^0 + \binom{3}{2}(x)^2(y)^1 + \binom{3}{1}(x)^1(y)^2 + \binom{3}{0}(x)^0(y)^3 \\ (1)(x^3) + (3)(x^2)(y) + (3)(x)(y^2) + (1)(y^3) \\ x^3 + 3x^2y + 3xy^2 + y^3 \end{array}$$

$$\text{b) } \begin{array}{c} (2x + y)^3 \\ \overbrace{\quad\quad\quad} \quad \overbrace{\quad\quad\quad} \quad \overbrace{\quad\quad\quad} \quad \overbrace{\quad\quad\quad} \\ \binom{3}{3}(2x)^3(y)^0 + \binom{3}{2}(2x)^2(y)^1 + \binom{3}{1}(2x)^1(y)^2 + \binom{3}{0}(2x)^0(y)^3 \\ (1)(2x)^3 + (3)(2x)^2(y) + (3)(2x)(y^2) + (1)(y^3) \\ 8x^3 + 12x^2y + 6xy^2 + y^3 \end{array}$$

$$\text{c) } \begin{array}{c} (2x - y)^4 \\ \overbrace{\quad\quad\quad} \quad \overbrace{\quad\quad\quad} \quad \overbrace{\quad\quad\quad} \quad \overbrace{\quad\quad\quad} \quad \overbrace{\quad\quad\quad} \end{array}$$

$$\binom{4}{4}(2x)^4(-y)^0 + \binom{4}{3}(2x)^3(-y)^1 + \binom{4}{2}(2x)^2(-y)^2 + \binom{4}{1}(2x)^1(-y)^3 + \binom{4}{0}(2x)^0(-y)^4$$

$$(1)(2x)^4 + (4)(2x)^3(-y) + (6)(2x)^2(-y)^2 + (4)(2x)(-y)^3 + (1)(-y)^4$$

$$16x^4 - 32x^3y + 24x^2y^2 - 8xy^3 + y^4$$

d)

$$(3x + 2y)^4$$

$$\binom{4}{4}(3x)^4(2y)^0 + \binom{4}{3}(3x)^3(2y)^1 + \binom{4}{2}(3x)^2(2y)^2 + \binom{4}{1}(3x)^1(2y)^3 + \binom{4}{0}(3x)^0(2y)^4$$

$$(1)(3x)^4 + (4)(3x)^3(2y) + (6)(3x)^2(2y)^2 + (4)(3x)(2y)^3 + (1)(2y)^4$$

$$81x^4 + 108x^3y + 216x^2y^2 + 96xy^3 + 16y^4$$

13.

$$\binom{10}{3}(2x)^3(-5y)^7$$

$$(120)(8x^3)(-78125y^7)$$

$$-75\,000\,000x^3y^7$$